



## Charakteristika RÁMCOVÉHO TÉMATU DIZERTAČNÍ PRÁCE uchazeče o studium v doktorském studijním programu P1801 „Informatika“

(Tento dokument je nutno předložit zároveň s přihláškou)  
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Oddíl A	Základní údaje	
1 Uchazeč o studium Bc. Václav Blažej	2 Navrhovaný školitel RNDr. Tomáš Valla, Ph.D.	
3 Navrhovaný předběžný název dizertační práce Algorithmic game theory		
Oddíl B	Charakteristika a zaměření dizertační práce	
4 Charakteristika a zaměření dizertační práce <p>Since building its foundations by Nash, von Neuman, Smith and others in the 50's, the field of the classical game theory has developed into a broad and well-studied branch of mathematics, with many application in social sciences like economy, biology, politics and computer science. This theory aims to mathematically model the behaviour of participants (players) of a certain competitive process. Recently, mostly because of the rapid growth of the Internet and its massive online auctions and online business, a large attention has been turned on the algorithmic aspects of the game theory.</p> <p>When we are talking about a game, or game setting of certain process, we consider there are selfish and greedy entities, which tend to compete each other. It is much harder to control and manoeuvre this environment, when compared to a setting with a central authority which may impose any demand on the environment and all entities. Among such "anarchy" settings are for example computer networks on the Internet (hardly anyone can dictate how should the connected computers behave, but still we would like to impose certain protocols and recommended behaviour), democratic economies (we need to preserve some freedom of subjects on the market, but we would still like to collect taxes, punish unfair traders and manipulate the whole economics in a certain direction), and many other examples.</p> <p>Note, that reaching some desired global objective in the "anarchy" setting is therefore much more complicated, at least in general. The authority in the game setting does not have the right to directly control the actions of the players, the only stimulus the authority can apply is the setting of game rules and payoffs of the players according to their behaviour.</p> <p>A natural question arises: How good is the state of the game from the point of view of some global optimisation criteria? In such applications, we may proceed quantitatively by imposing a specific objective function, defined on the outcomes of the game, that numerically expresses the "social good" or "social cost" of an outcome.</p> <p>We seek a solution of the game where individual players act according to their incentives, greedily maximising their payoff. Probably the most popular way how to capture this concept is by the notion of a Nash equilibrium. This concept was introduced by Nash and has quickly attained the status of the central solution concept in game theory.</p>		

However, a game may have many equilibria with different objective function, some equilibria thus reach very good social outcomes, where other may be very inefficient. The most popular measure of the inefficiency of equilibria is the Price of Anarchy. The most interesting games for us are those in which the Price of Anarchy is close to 1. In such games, all equilibria are good approximations of an optimal outcome.

A Social Choice is an aggregation of the preferences of the players towards a common decision. Mechanism Design attempts implementing desired social choices in a strategic setting---assuming that the players (members of the society) each act rationally in a game theoretic sense.

Therefore, based on the previous thoughts, we formulate the general topic of this thesis: We study the complexity that often suddenly appears, when we consider the competitive version of a certain environment, process or behaviour.

We summarize the research goals as follows:

Establish new results in algorithmic game theory, mainly by:

- \* designing games where the result (usually Nash equilibria) correspond to globally desirable outcomes, mainly for combinatorial optimization problems, but not only,
- \* investigating its algorithmic and computational complexity aspects,
- \* investigating common phenomena of the original (i.e. usually optimization) problems and their game-theoretical formulations.

These research goals were already pursued in the diploma thesis of the applicant. Thesis was focused on the online Ramsey theory, which is a common field of both Ramsey theory and game theory. Thesis contains original results. First it shows an infinite family of trees, for which the online Ramsey number is asymptotically smaller than size-Ramsey number. Then it establishes upper bounds on online Ramsey numbers for cycles and  $k$ -subdivided trees, and proves the exact value of restricted online Ramsey number of triangles versus stars on connected graphs.

The future work of the applicant will build on results of the thesis, focusing on characterizing classes of graphs where online Ramsey number asymptotically differs from size-Ramsey number and investigating bounds of various classes of graphs. In the longer term the applicant will turn his attention algorithmic game theory, focusing on mechanism design, for example for combinatorial optimization problems.

**Oddíl C**

**Datum a podpis školitele a uchazeče o studium**

5 Datum a podpis školitele

6 Datum a podpis uchazeče o studium