A Simple Streaming Bit-parallel Algorithm for Swap Pattern Matching

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Exact pattern matching

In computer science, **pattern matching** is the act of checking a given **sequence** of tokens for the presence of the constituents of some pattern. In contrast to pattern recognition, the match usually has to be exact. The patterns generally have the form of either sequences or tree structures. Uses of pattern matching include outputting the locations (if any) of a pattern within a token sequence, to output some component of the matched pattern, and to substitute the matching pattern with some other token sequence (i.e., search and replace).

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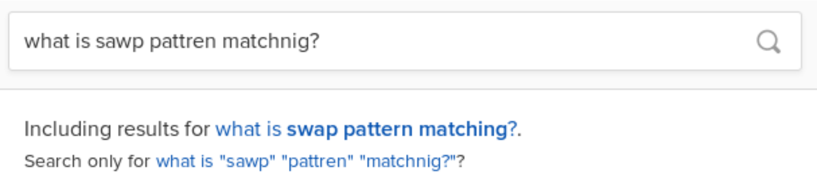

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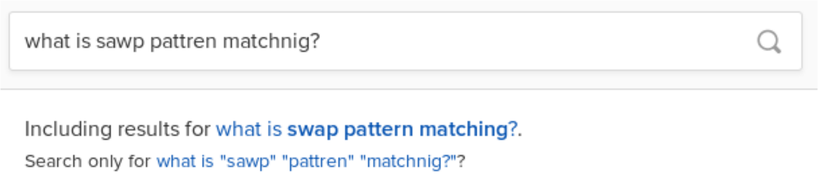

- we are allowed to swap adjacent symbols

Definition of Swap Matching problem

We search for occurrences of patterns in the text while allowing pattern to swap adjacent symbols.
We define swaps $\pi : \{1 \dots n\} \to \{1 \dots n\}$ in the pattern S such that:

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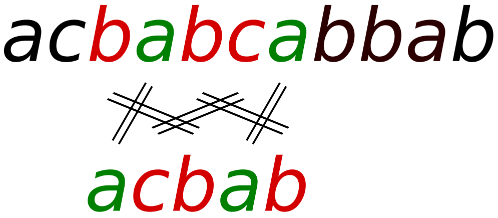

Swap Matching example

Search for abba in text abbabaaababbbaaabbabaaabbbbaab:

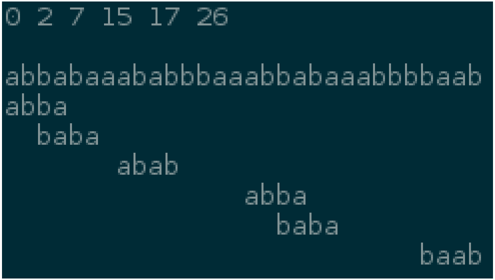

Figure: found occurrences of abba in the text

History

- 1995 - Swap Matching problem was announced as open problem [Muthukrishnan, CPM 95]
- 1997 - first solution using FFT, $O(nm^{\frac{1}{2}} \log m)$ [Amir et al., J. Algorithms]
- 2008 - first non-FFT algorithm, using bit-parallelism [Iliopoulos and Rahman, SOFSEM 2008] $O((n+m)\log m)$
- 2009 - Cross Sampling algorithm which solves the problem in $O(n)$ for short patterns [Cantone and Faro, SOFSEM 2009]
- 2013 - new model using reactive automata and solution with $O(n)$ complexity for short patterns [Faro, PSC 2013]
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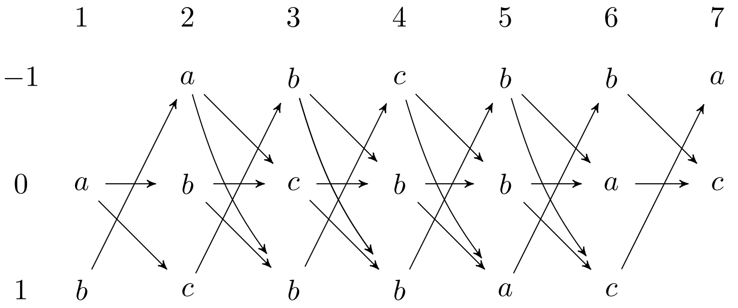
- Graph represents all patterns which are feasible.



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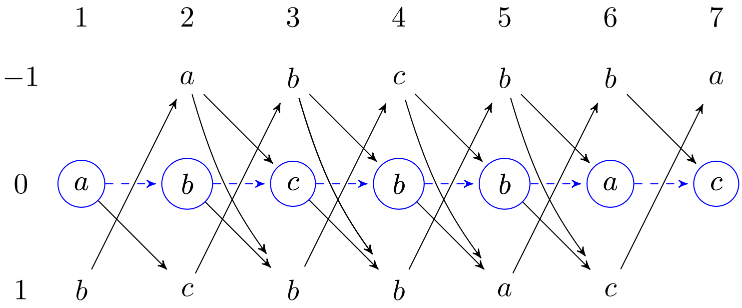

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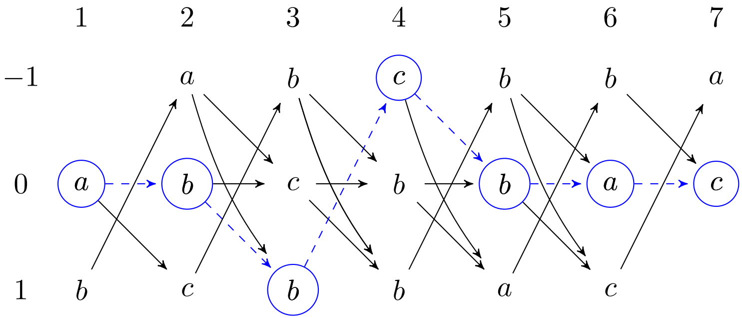

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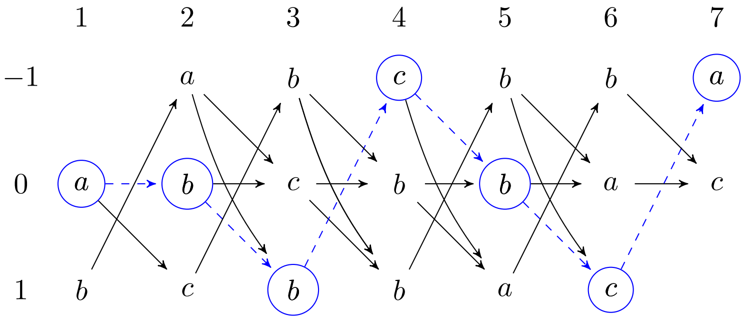

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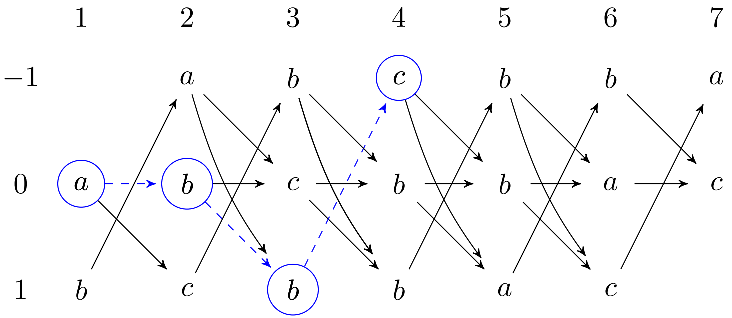
- Each path from first to last column is one such pattern.
- The *signal* is information that a path partially matches.



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- uses the graph theoretical model,
- takes input as stream of symbols,
- bitwise parallelism of machine instructions,
- can be implemented using only $7 + |\Sigma|$ memory cells.

Our algorithm

We use the bitwise representation and operations to simulate signal propagation through the model.

- Represent each row with bit array.
- Use *shift* and *or* to move signal.
- Use *and* to filter our signal.

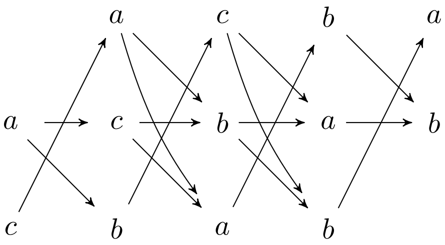

Figure: Example run for $P = acbab$ and $T = acabab$

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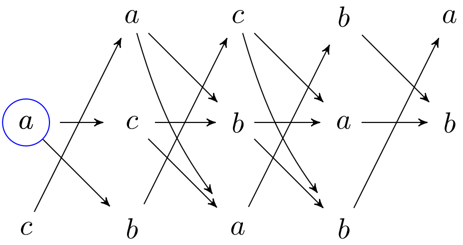

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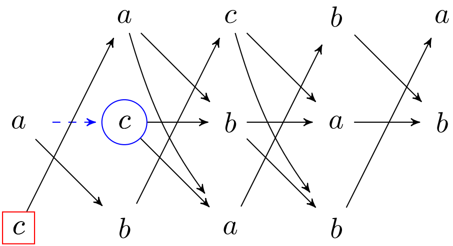

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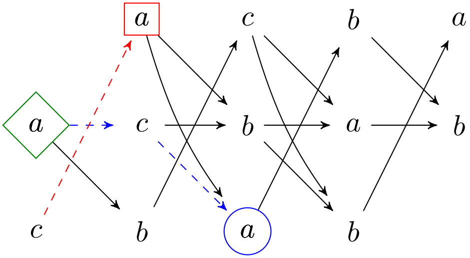

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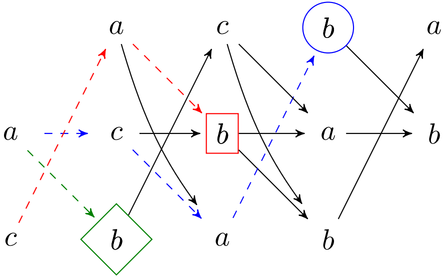

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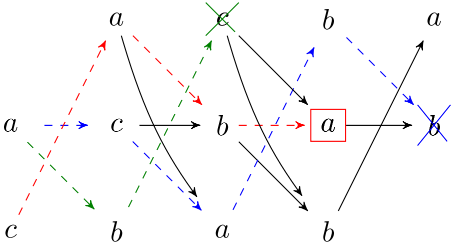

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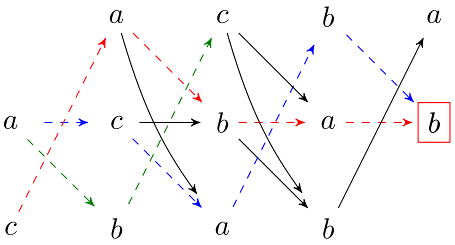

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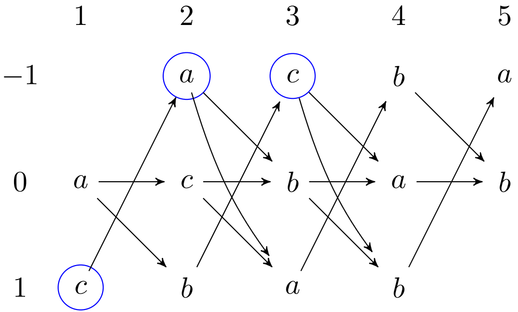

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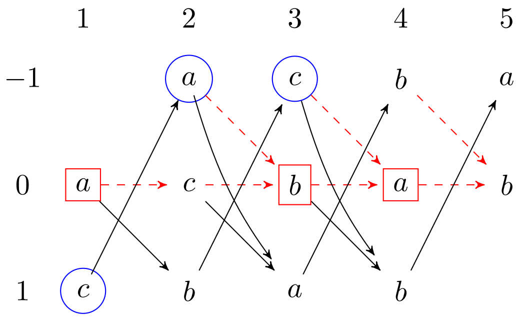
- for row 0 shift and make *or* of rows -1 and 0



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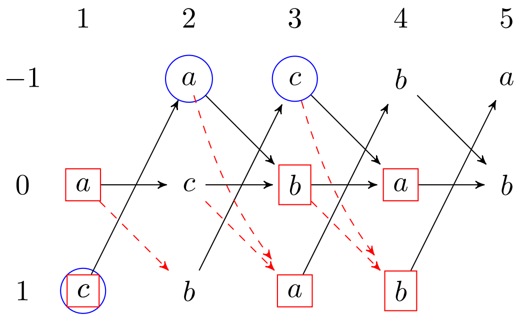

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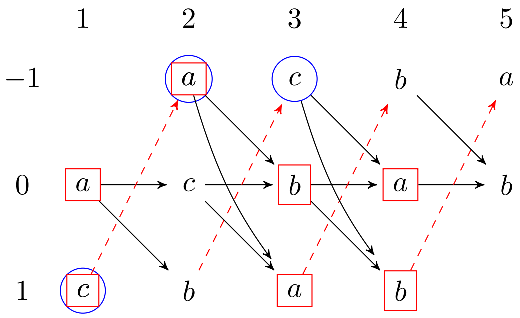
- for row -1 shift and add row 1



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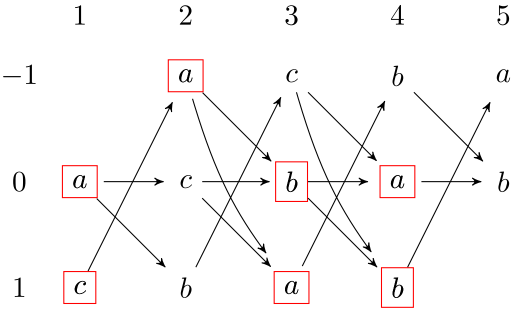

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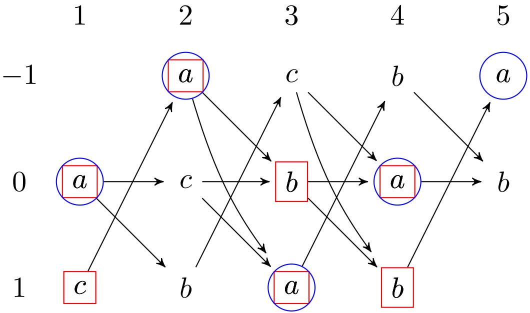

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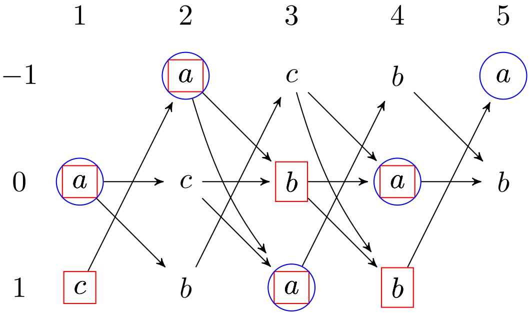

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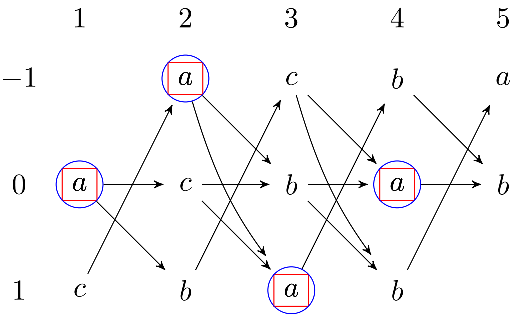

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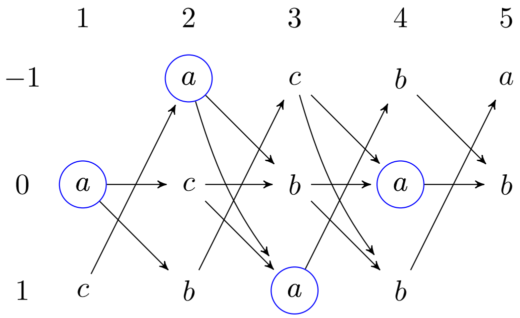

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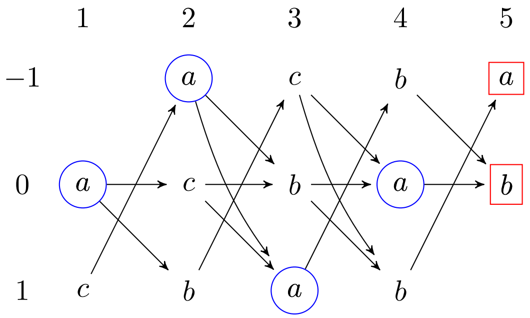
Step 3 – check result

- check if there is a signal in the last column



Properties of our algorithm

For pattern of length m, text n and word size w (using the word-Ram model) we have

- time complexity

$$O(\lceil \frac{m}{w} \rceil (|\Sigma| + n) + m),$$

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- If $m \leq w$ we get $O(|\Sigma| + m + n)$ time and $O(|\Sigma|)$ space.

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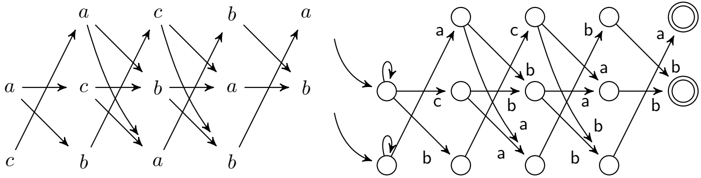

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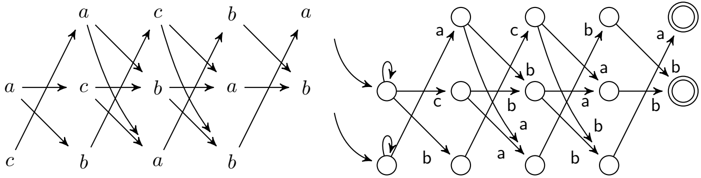

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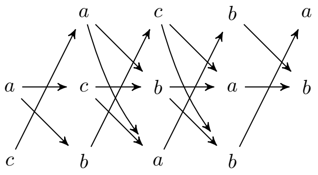
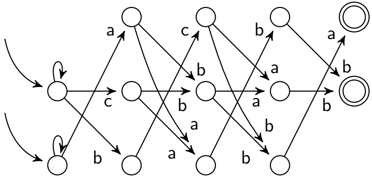
- Any time it reaches final state it reports an occurrence and continues reading input.





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Theorem

There is an infinite family F *of patterns such that any deterministic finite automaton* A_P *accepting the language* $L_S(P) = \{u\pi(P) \mid u \in \Sigma^*, \pi$ *is a swap permutation for* $P\}$ *for* $P \in F$ *has* $2^{\Omega(|P|)}$ *states.*

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Length if these patterns is $|4 + 5k|$ a $k \in \{1, 2, \dots\}$.

$P = T_0$	$acccabcccabccc$
T_1	$acccbacccabccc$
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Table: All strings for $k = 2$.

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If the automaton is in the same state after reading strings T_i, T_j such that $T_i \neq T_j$, then there exists such a suffix S such that $T_i.S \in A$ and $T_j.S \notin A$.

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Our results:

- new algorithm for Swap Matching
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