ON THE PARAMETERIZED COMPLEXITY OF EULERIAN STRONG COMPONENT ARC DELETION

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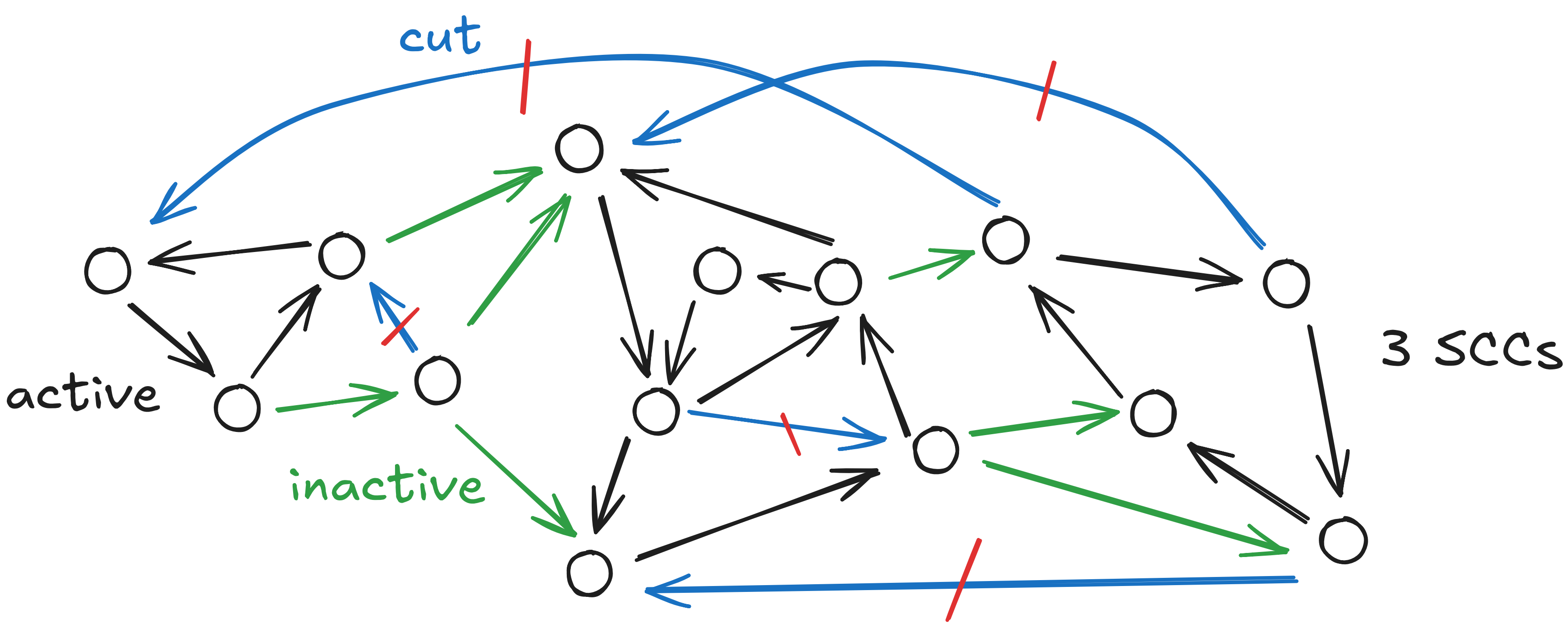
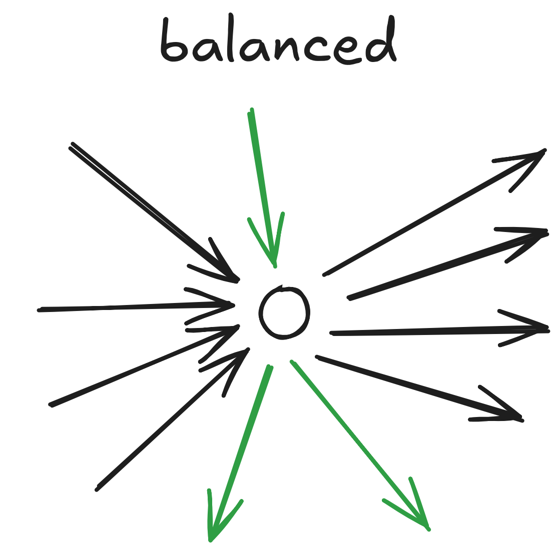
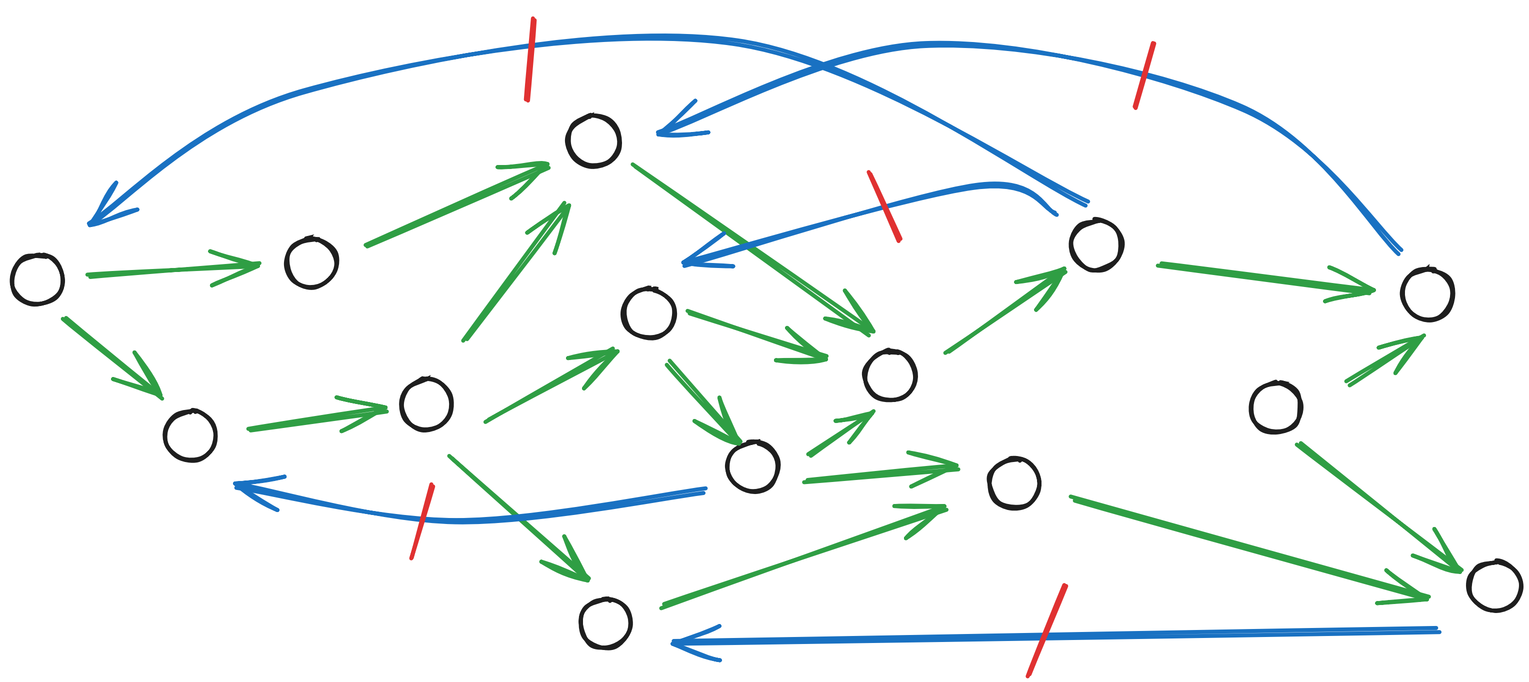




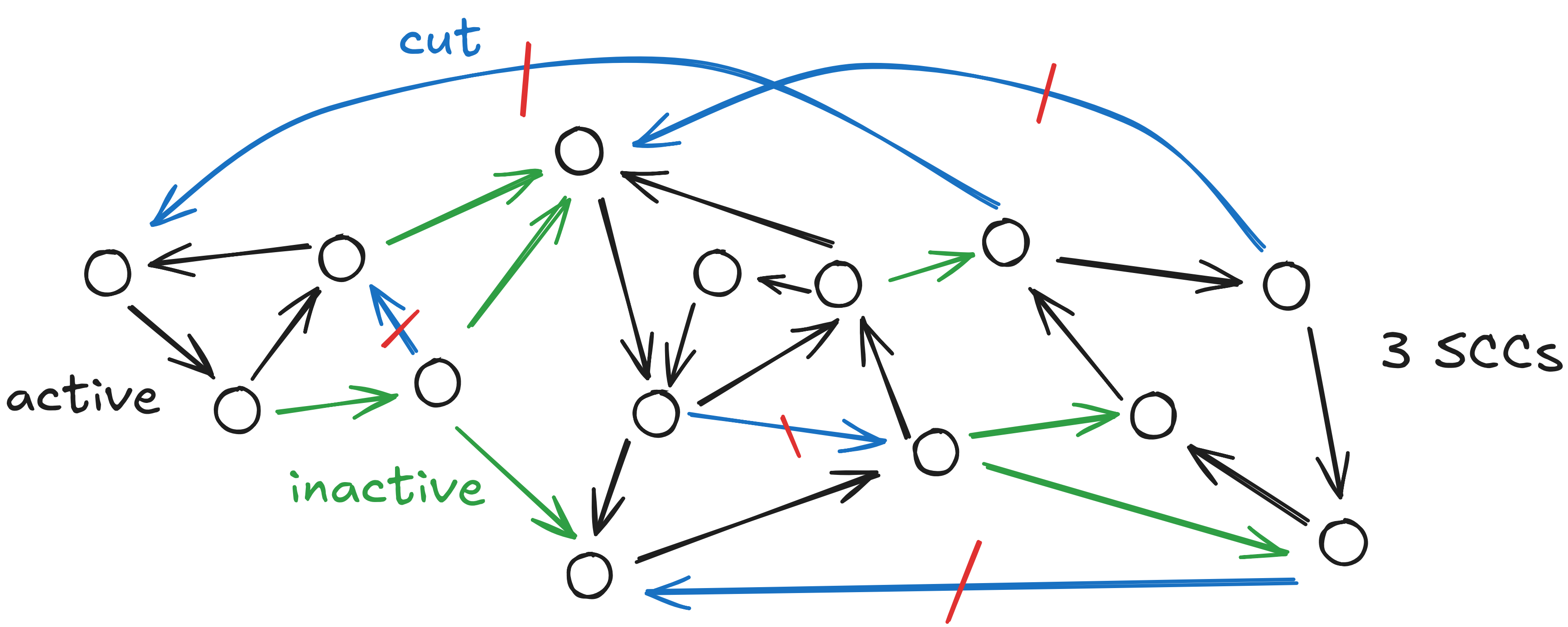
FEEDBACK ARC SET

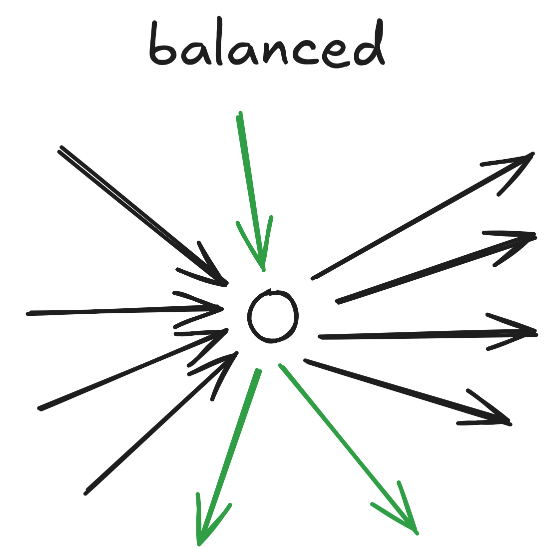
- **Input:** directed graph G, integer k
- **Output:** Is there a set of arcs S, $|S| \leq k$, such that every **strongly connected component** of $G - S$ has size one?



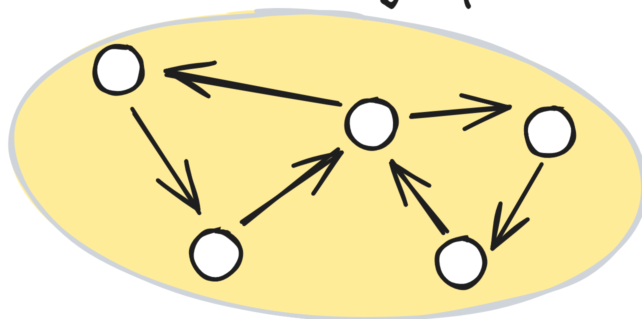
EULERIAN STRONG COMPONENT ARC DELETION (ESCAD)

- **Input:** directed graph G, integer k
- **Output:** Is there a set of arcs S, $|S| \leq k$, such that every strongly connected component of $G - S$ ~~has size one~~ is **Eulerian**?



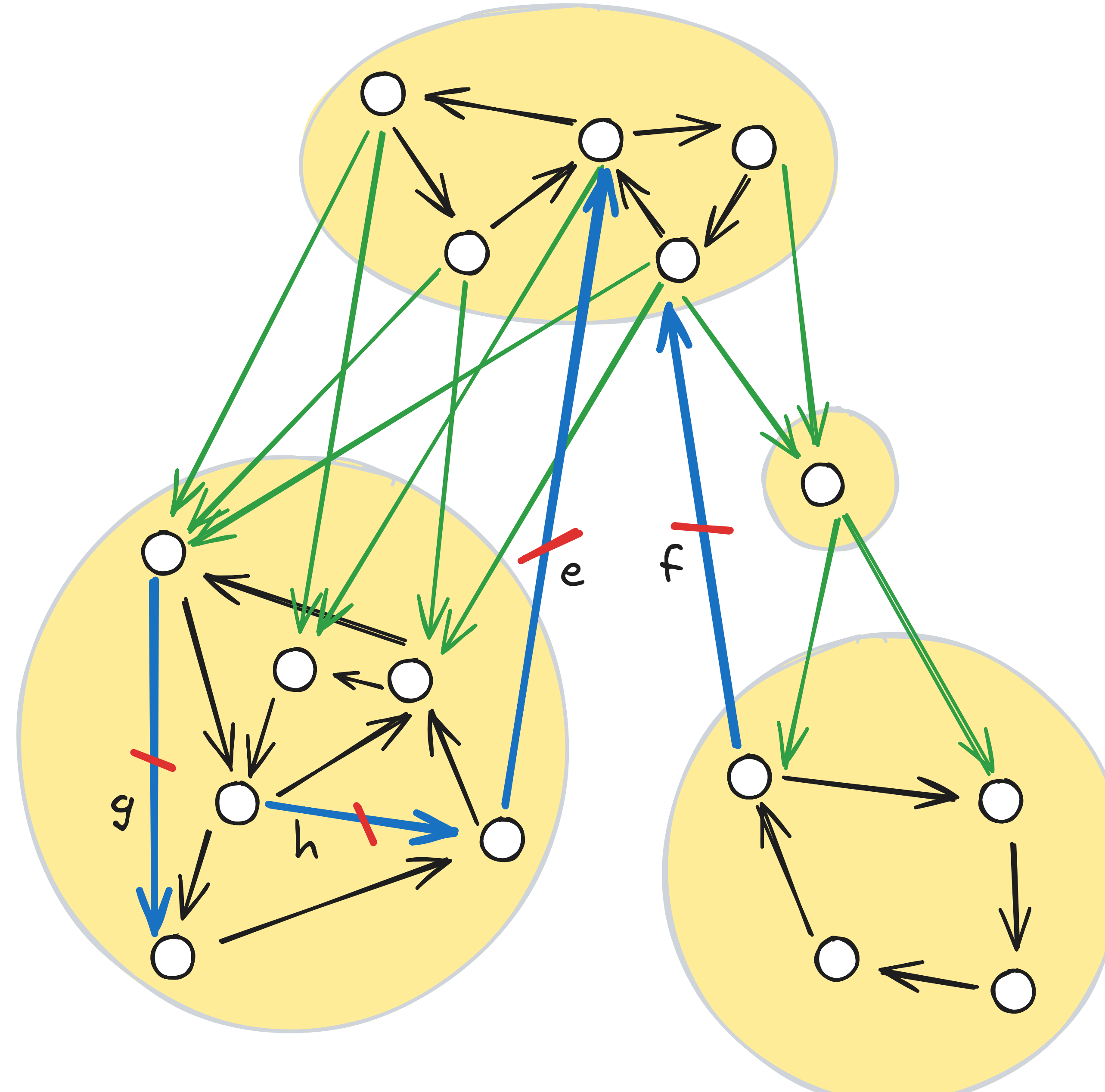


Eulerian graph



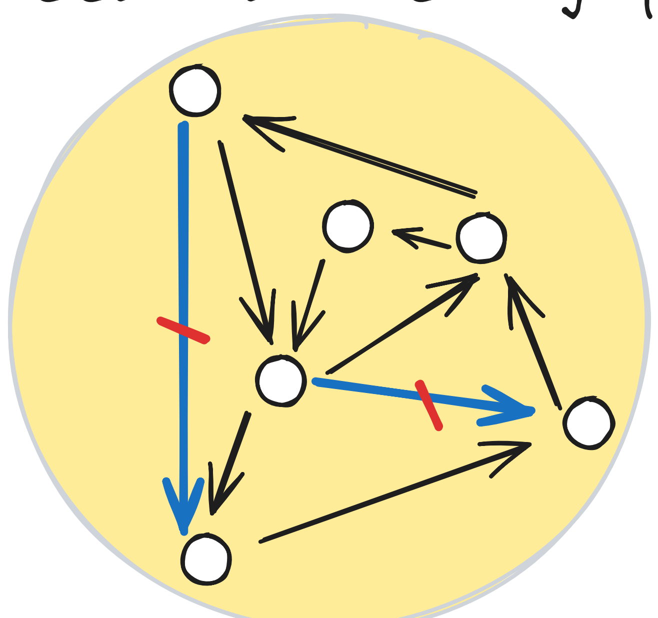
solution size 0

ESCAD



solution size 4

deletion to Eulerian graph



solution size 2

PREVIOUS WORK

same problem but removing **vertices** instead

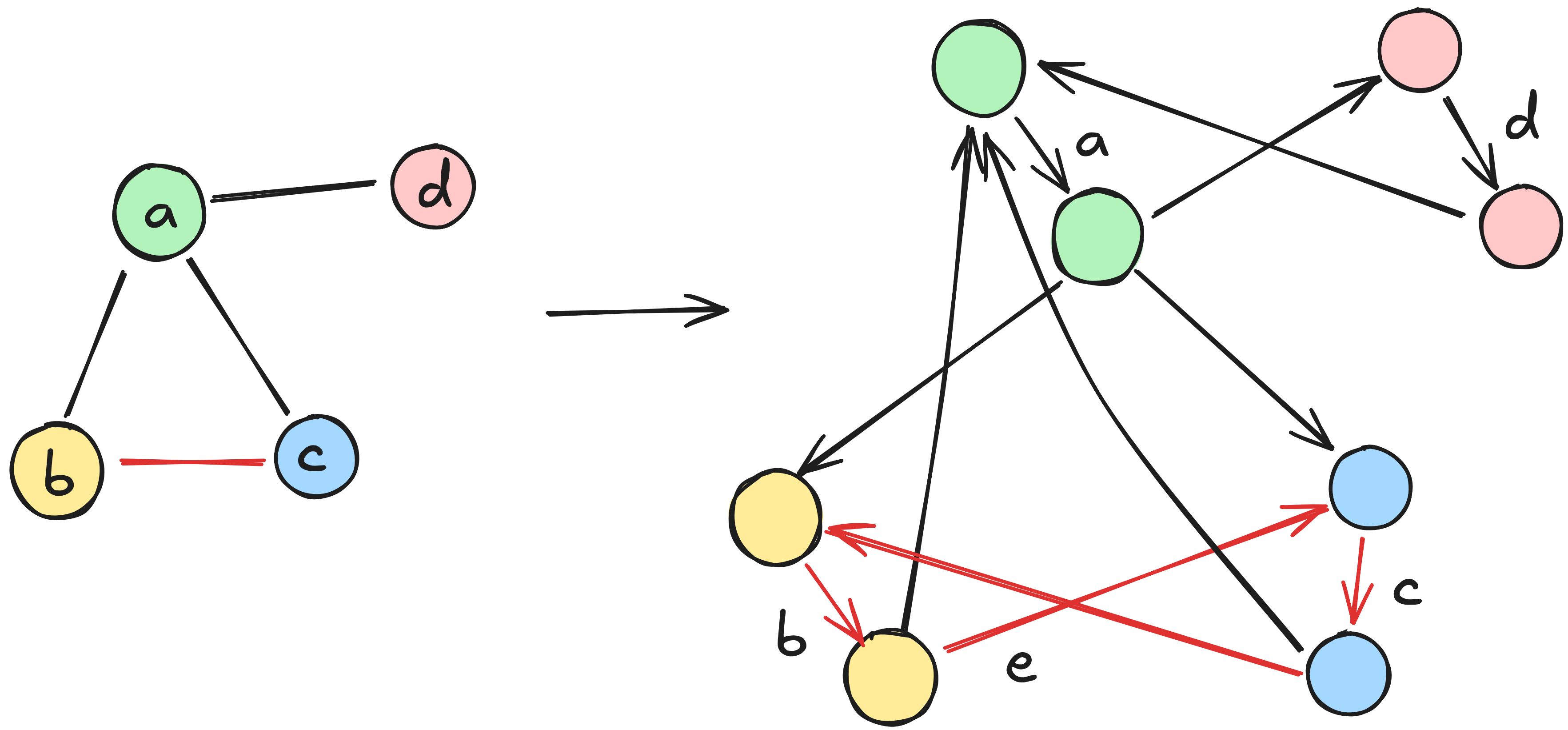
- Göke, Marx, Mnich; *Discret. Optim. (2022)*
  NP-hardness of the vertex deletion variant

requiring $G - S$ to be Eulerian (balanced + **connected**)

- Cygan, Marx, Pilipczuk, Pilipczuk, Schlotter; *Algorithmica (2012)*
  FPT algorithm for arc deletion to Eulerian graphs
- Goyal, Misra, Panolan, Philip, Saurabh; *J. Comput. Syst. Sci. (2018)*
  single-exponential FPT algorithm arc deletion to Eulerian graphs

FEEDBACK ARC SET IS NP-HARD

by a reduction from vertex cover



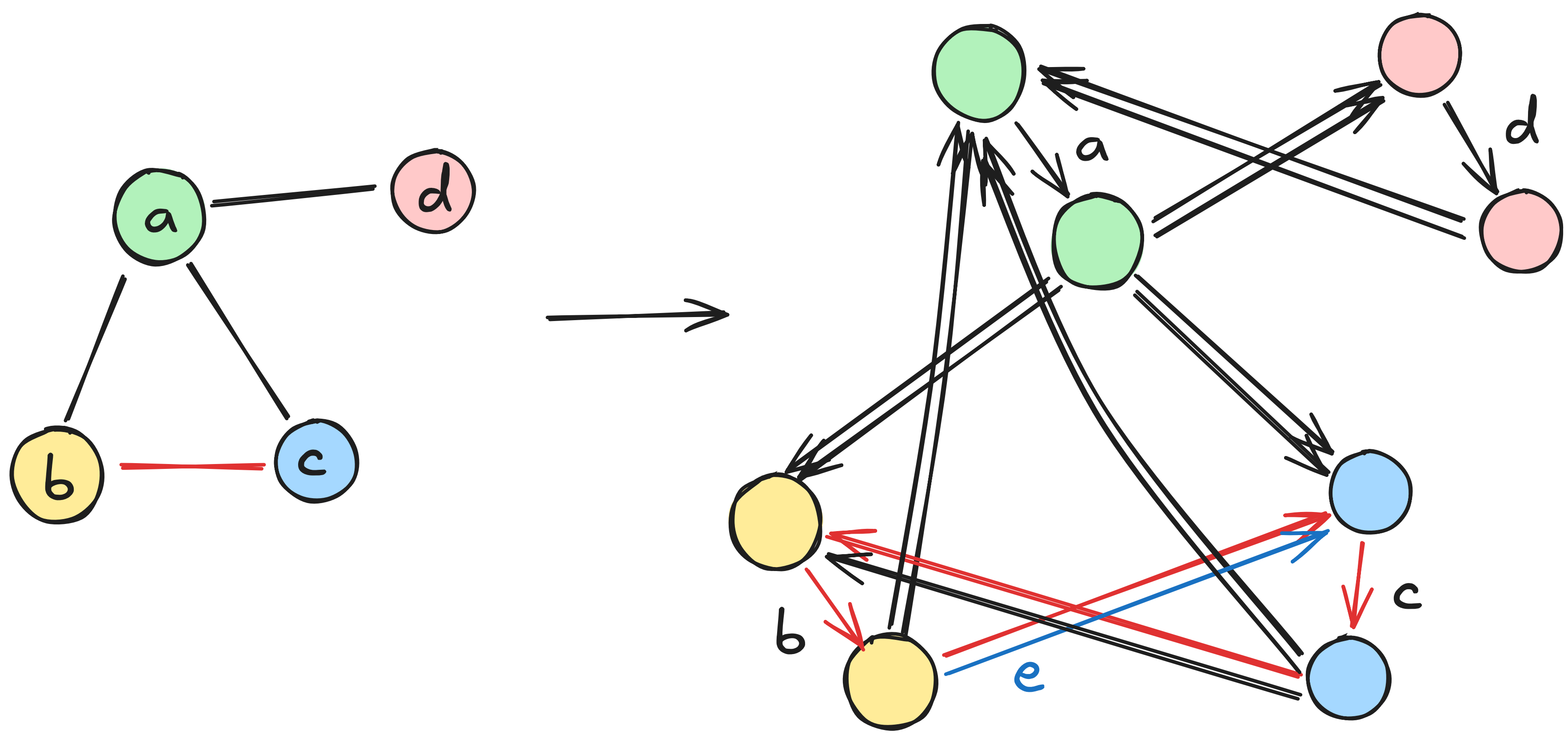
NODE COVER $\propto$ FEEDBACK ARC SET

$$V = N' \times \{0,1\}$$

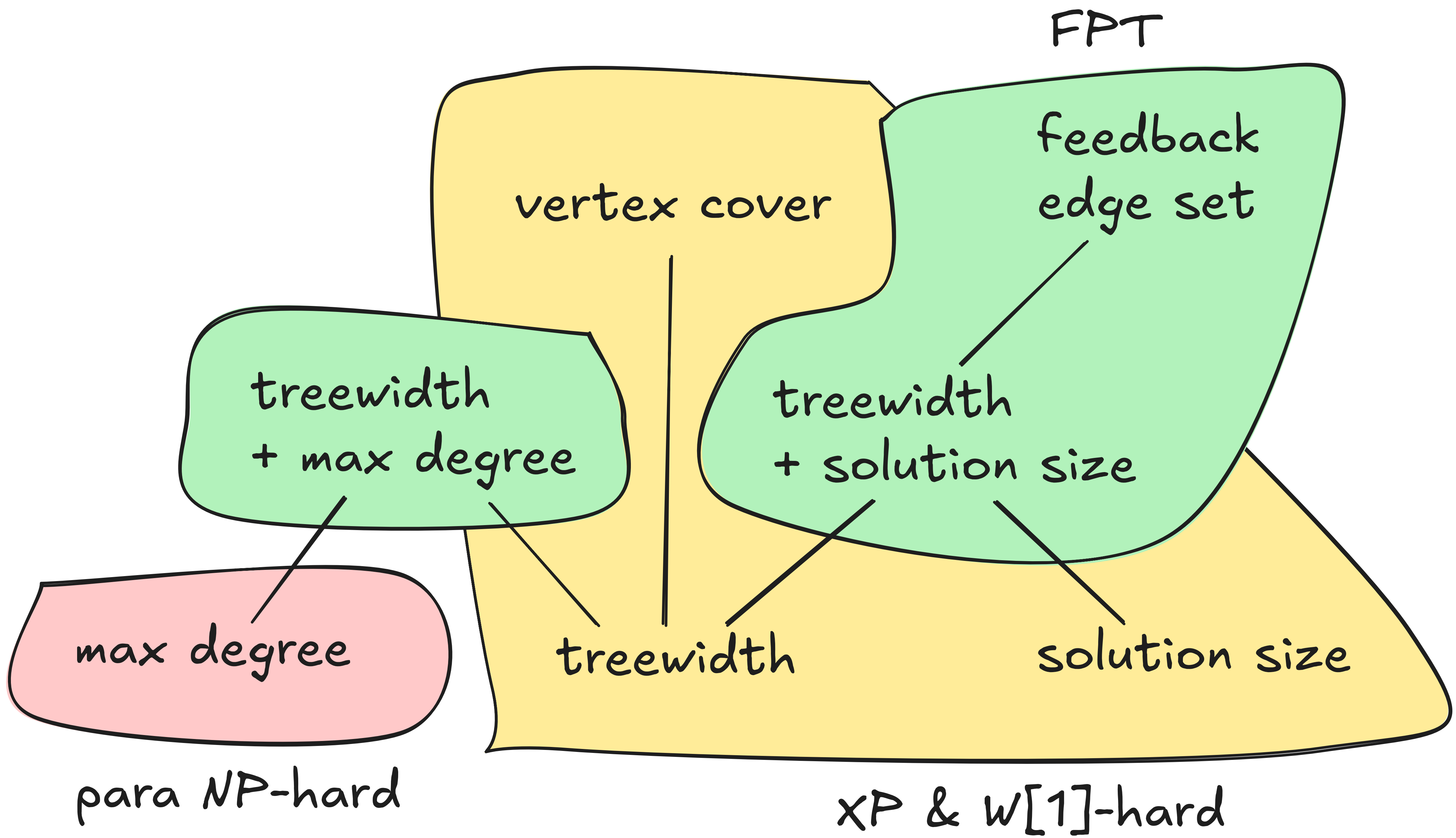
$$E = \{\langle\langle u,0\rangle,\langle u,1\rangle\rangle \mid u \in N'\} \cup \{\langle\langle u,1\rangle,\langle v,0\rangle\rangle \mid \{u,v\} \in A'\}$$

$$k = \ell.$$

EULERIAN STRONG COMPONENT ARC DELETION IS NP-HARD FOR MAXIMUM DEGREE 7

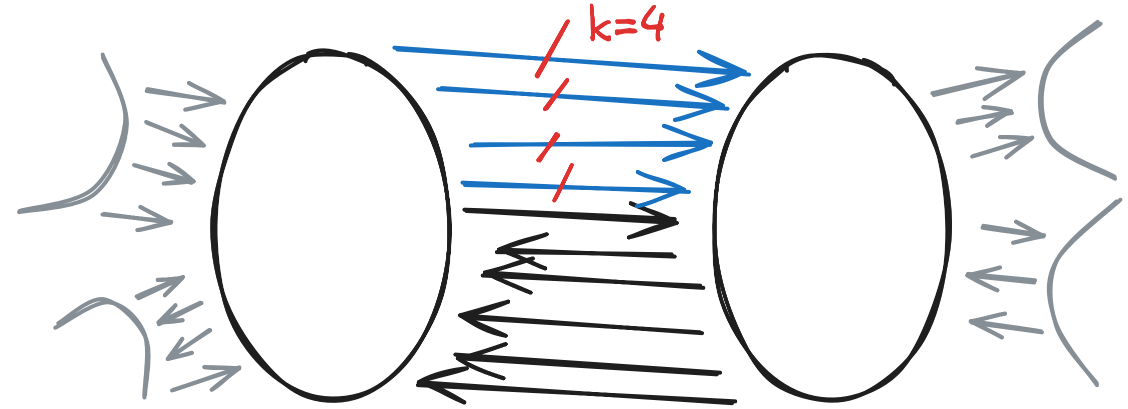


vertex cover is NP-hard for max degree 3

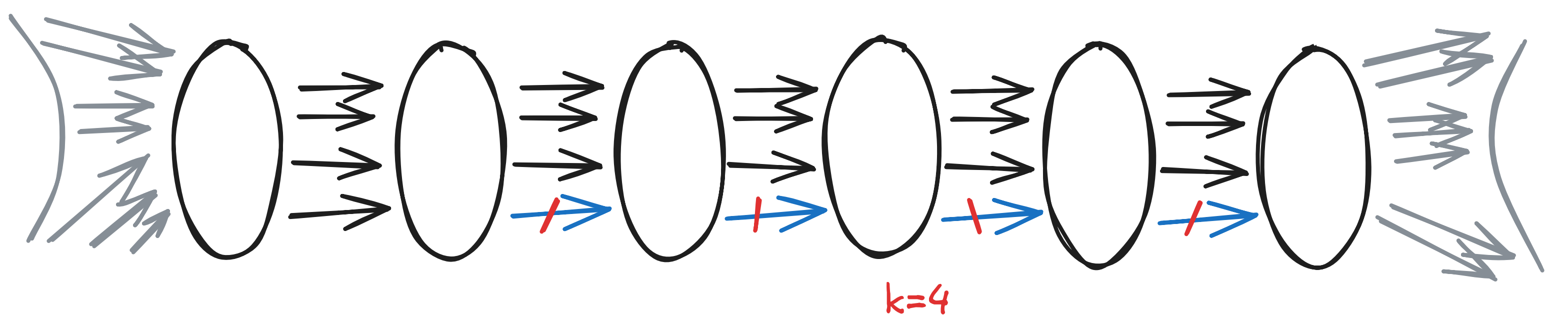


* *parameters of the underlying undirected graph*

well connected → unseparable

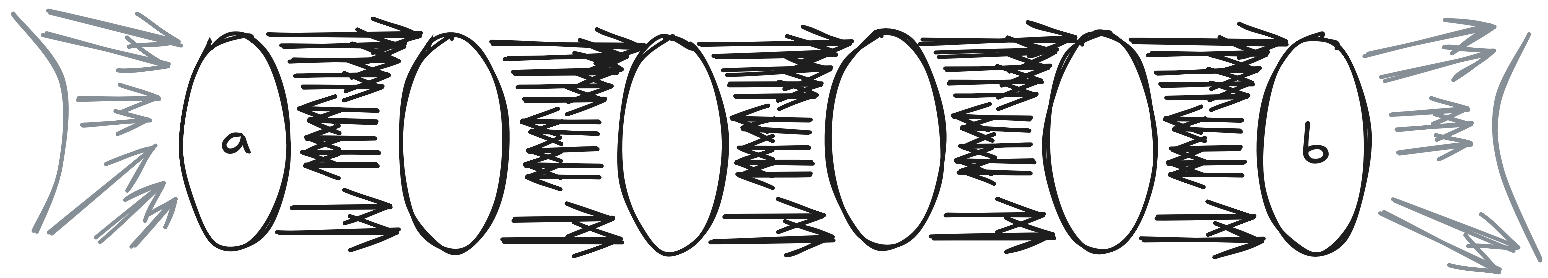


long paths → cannot be thinned



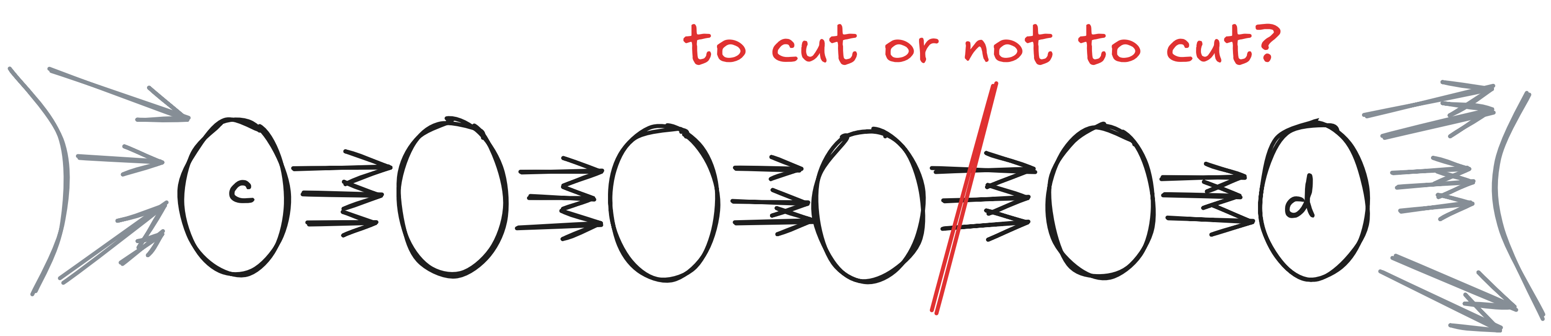
UNDELETABLE EDGES

solution has none of its edges; creates imbalance

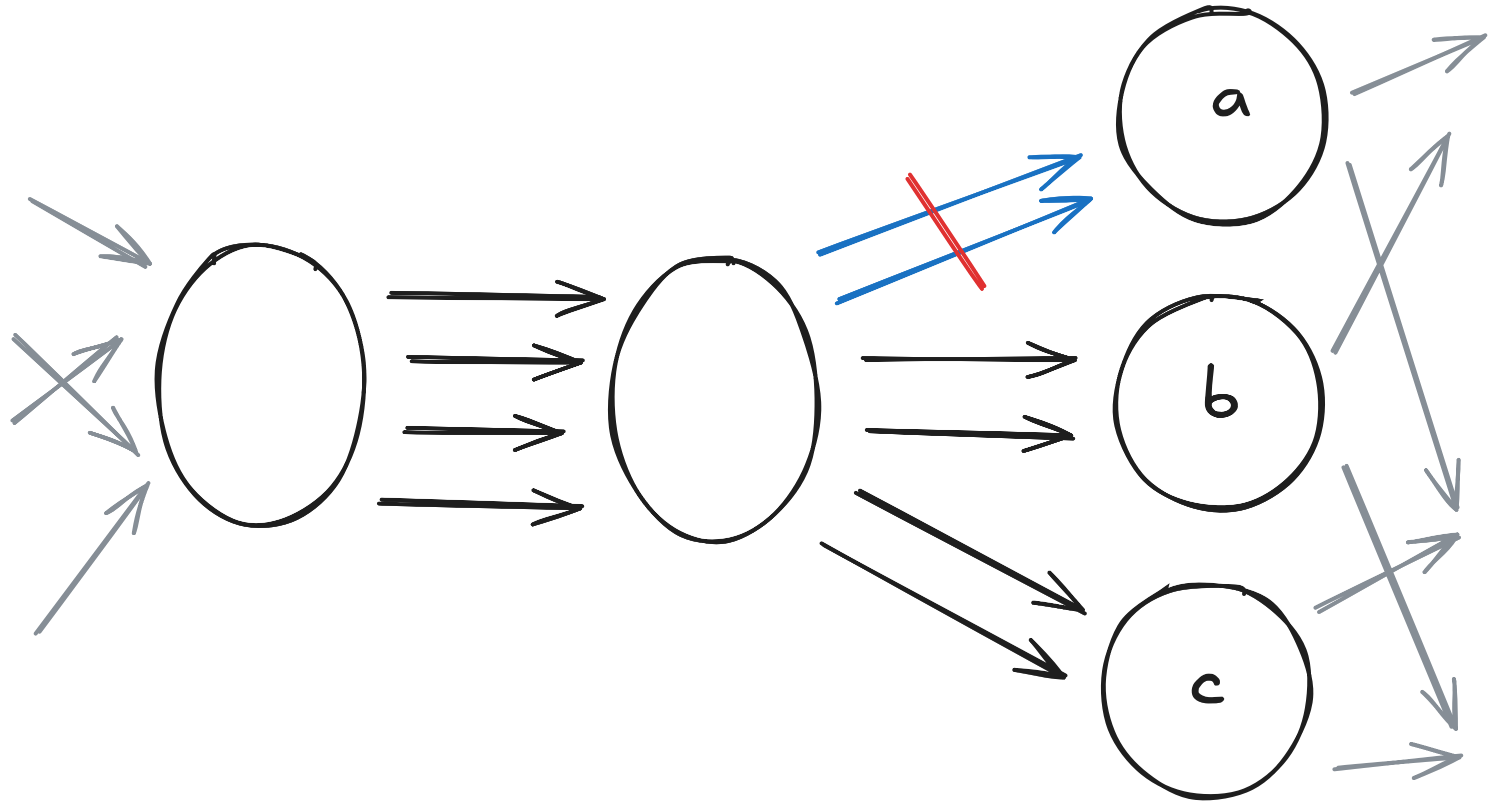


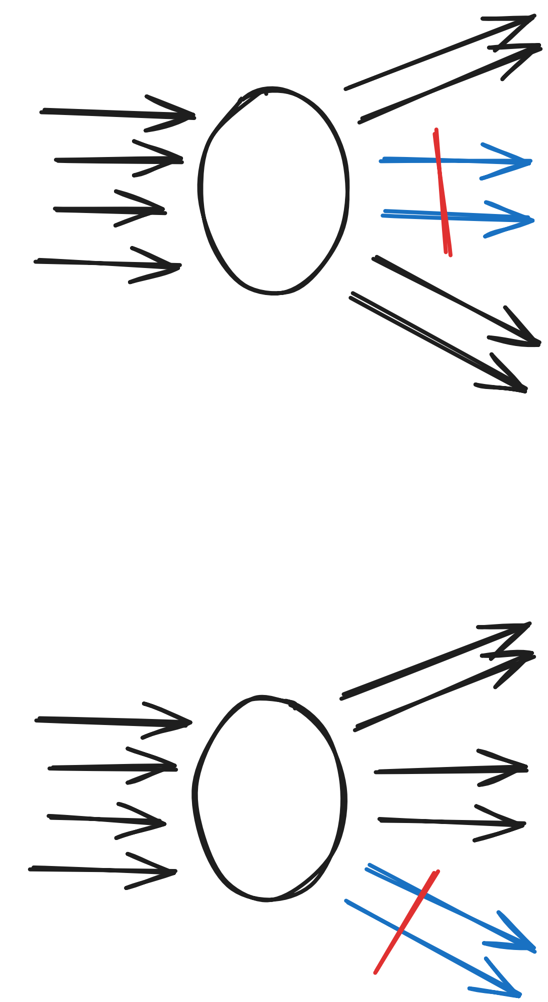
CUT ALL-OR-NONE EDGES

solution can only cut it

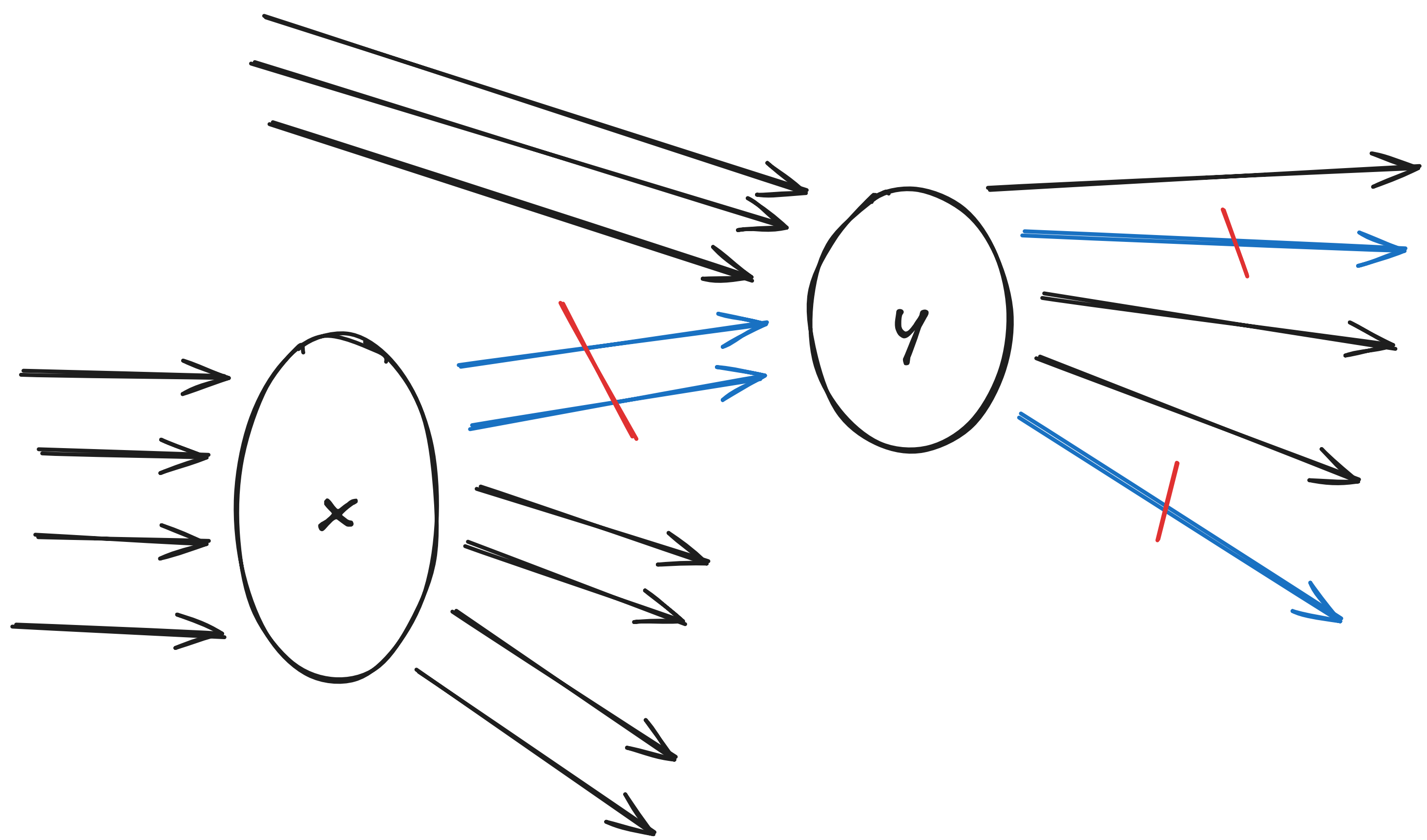


CHOICE GADGET



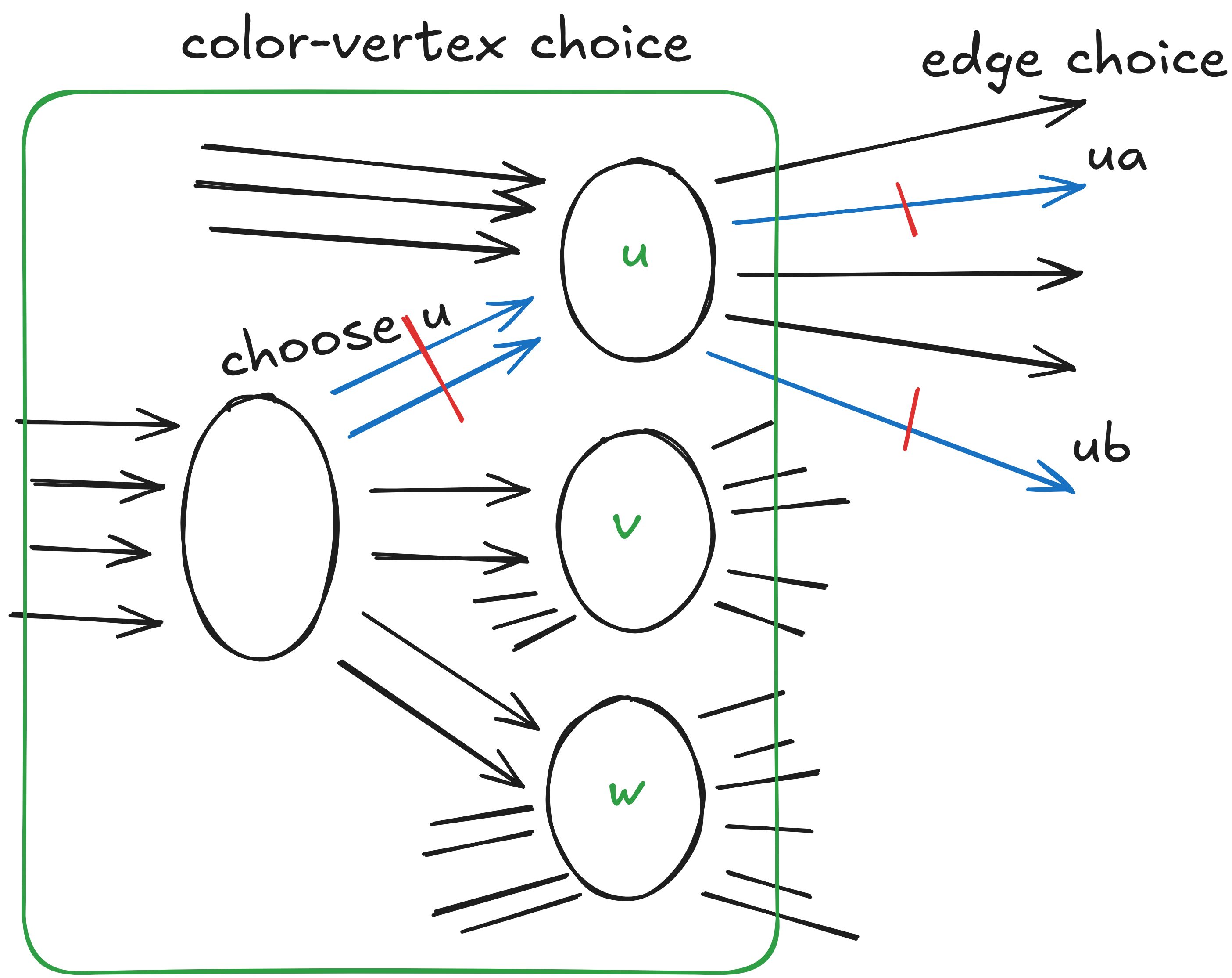


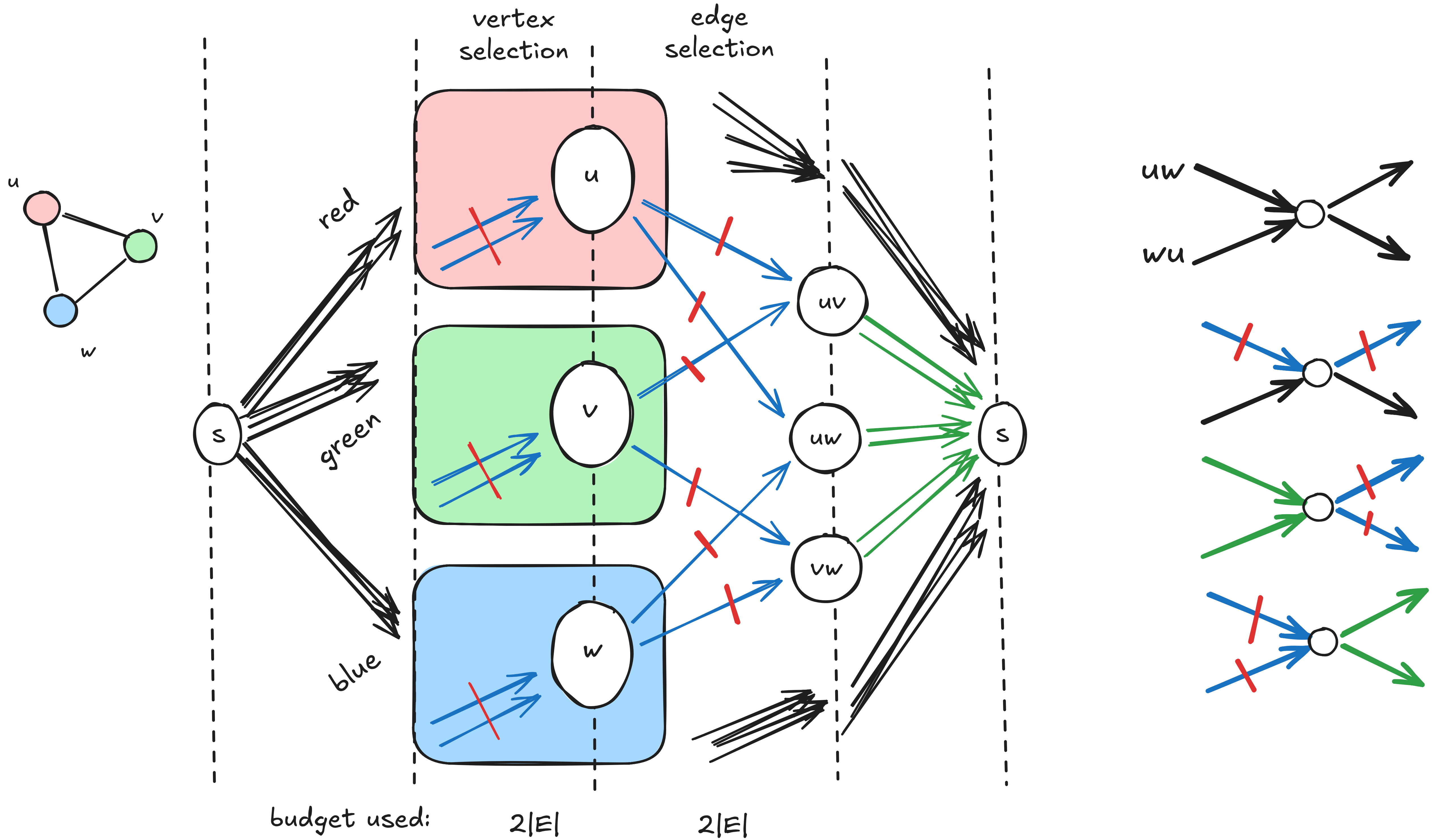
PROPAGATING CHOICES



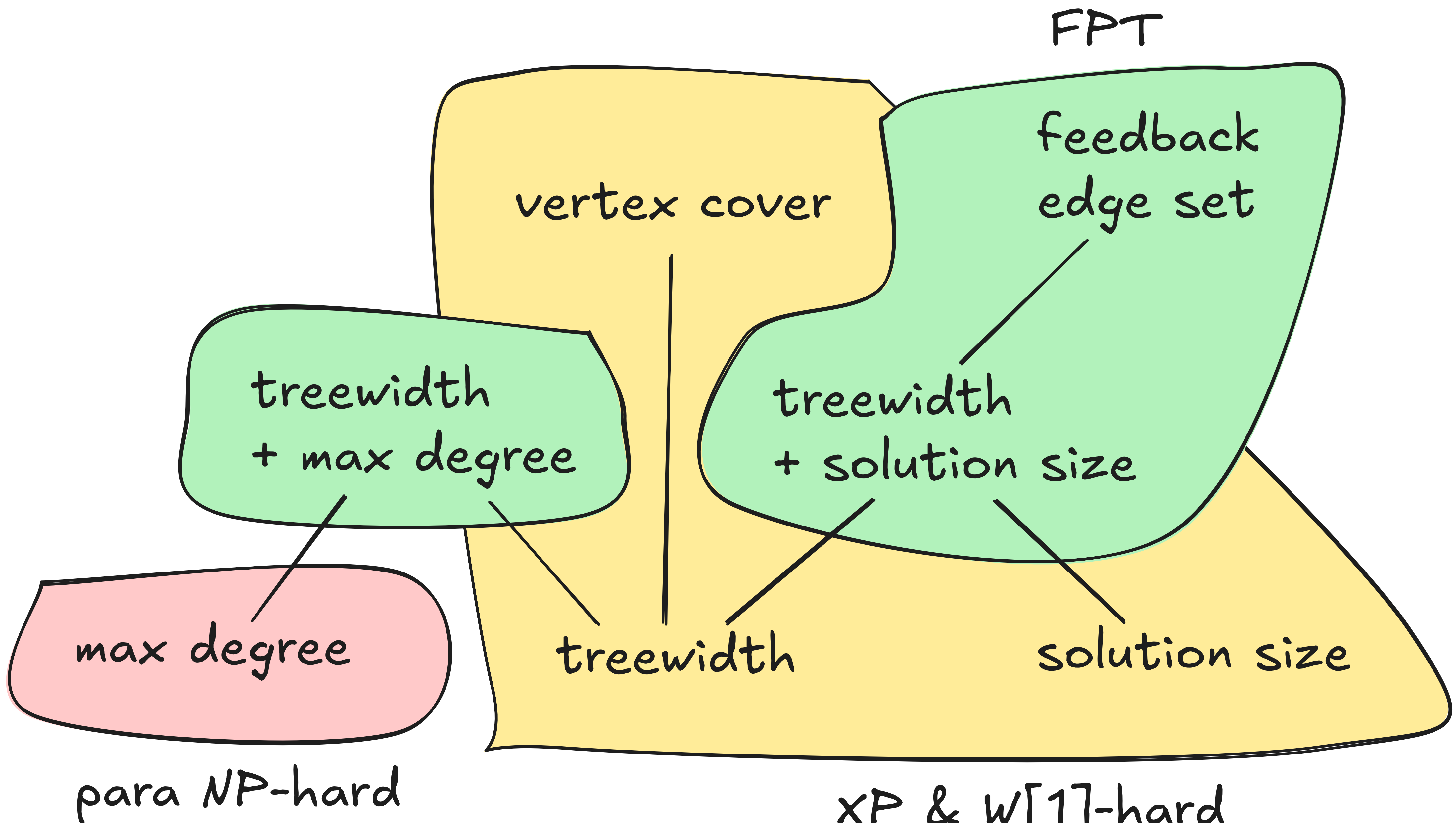
ESCAD IS $W[1]$-HARD BY THE SOLUTION SIZE

by a reduction from multicolored clique





vertex
selection
edge
selection
u
v
w
red
green
blue
s
u
v
w
uv
uw
vw
s
uw
wu
budget used:
2|E|
2|E|

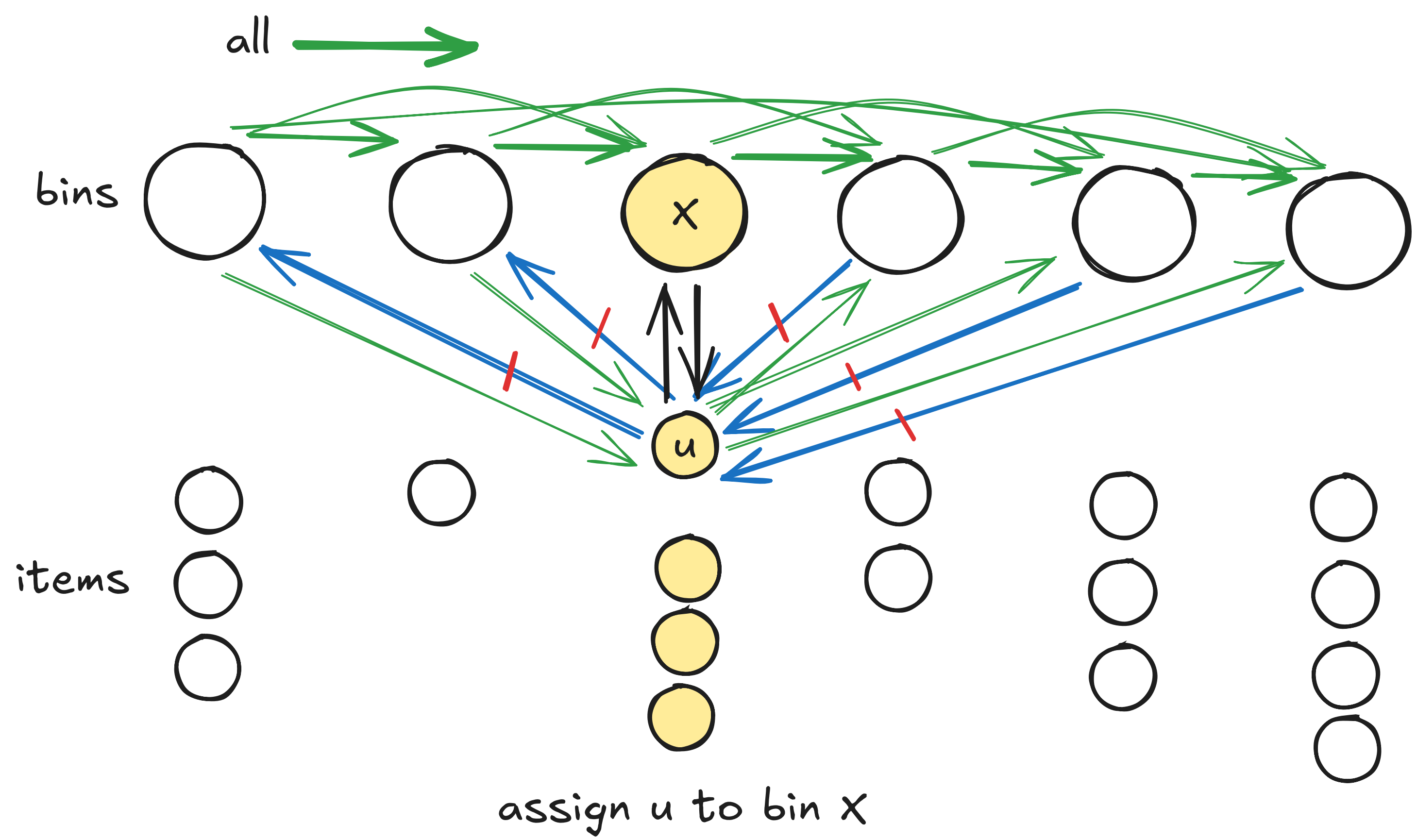

FPT
vertex cover
feedback edge set
treewidth + max degree
treewidth + solution size
max degree
treewidth
solution size
para NP-hard
XP & W[1]-hard

ESCAD IS $W[1]$-HARD BY VERTEX COVER

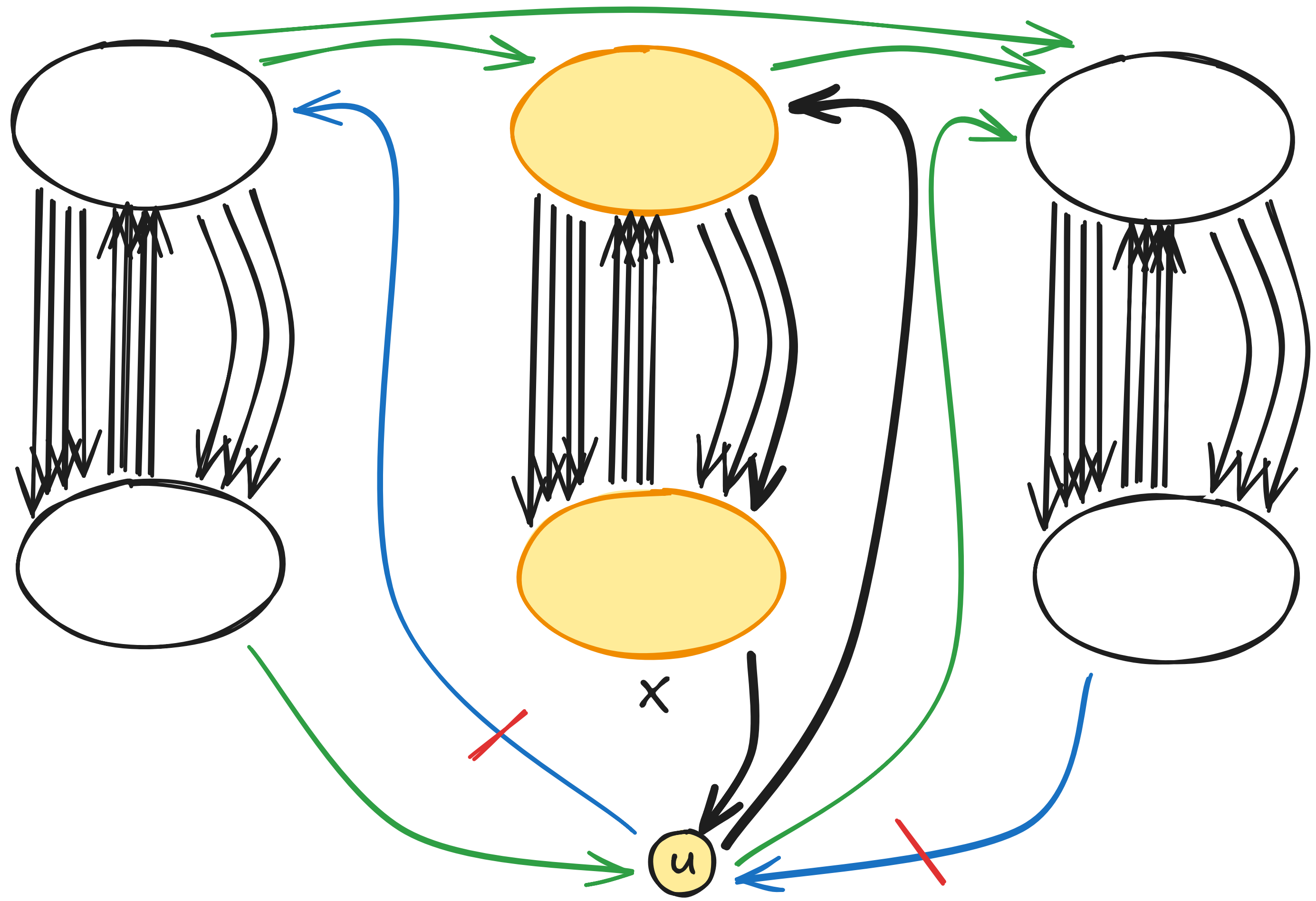
Unary Bin Packing

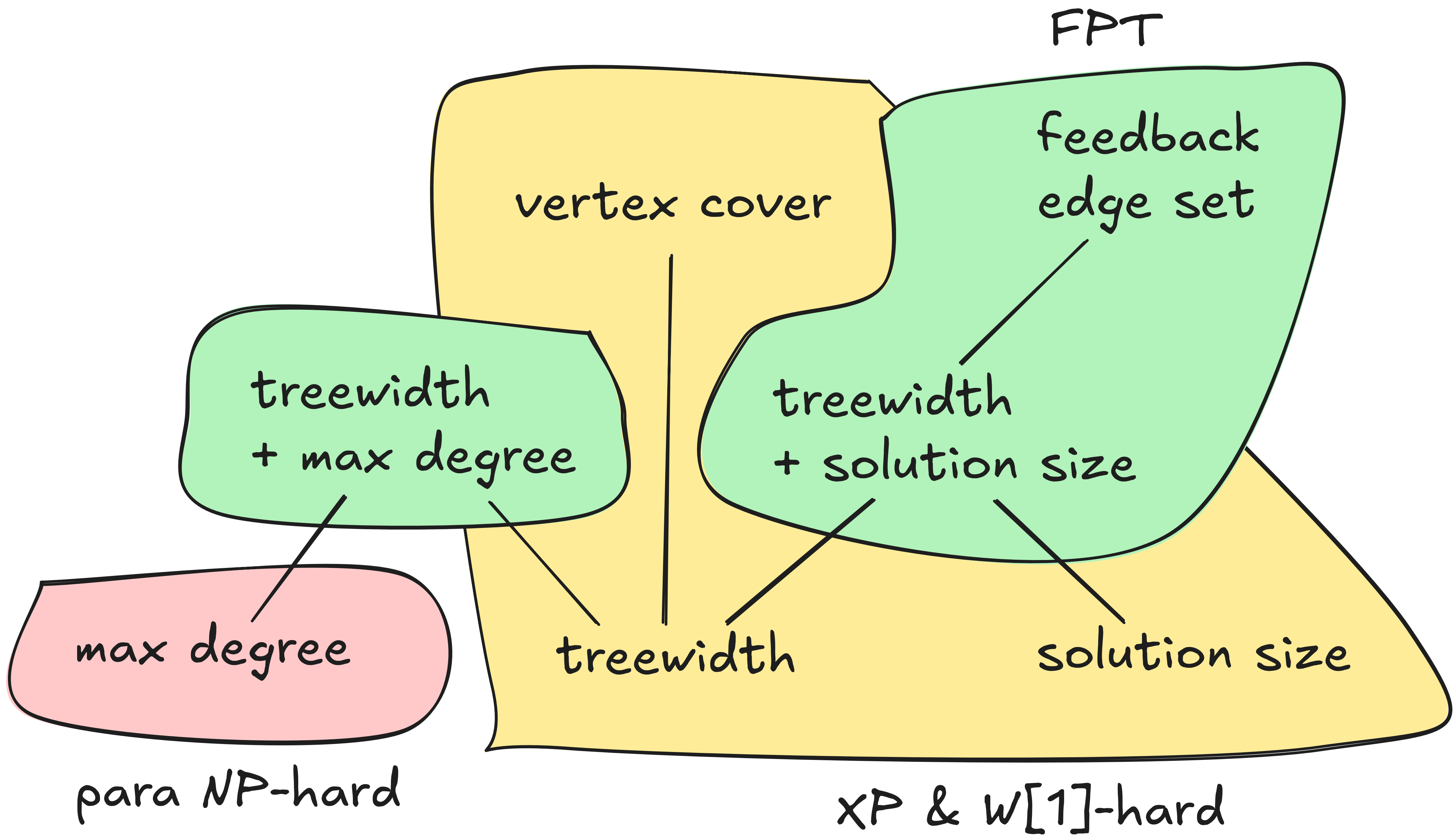
- **input:** A set of positive integer item sizes $x_1, \ldots, x_n$ encoded in unary, a pair of integers h and b.
- **output:** Is there a partition of $[n]$ into h sets $J_1, \ldots, J_h$ such that $\sum_{\ell \in J_j} x_\ell \leq b$ for every $j \in [h]$?
- Jansen, Kratsch, Marx, and Schlotter; *Comput. Syst. Sci., (2013)* Unary Bin Packing is $W[1]$-hard by the number of bins b

ESCAD IS $W[1]$-HARD BY VERTEX COVER



ESCAD IS $W[1]$-HARD BY VERTEX COVER




FPT
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treewidth
+ max degree
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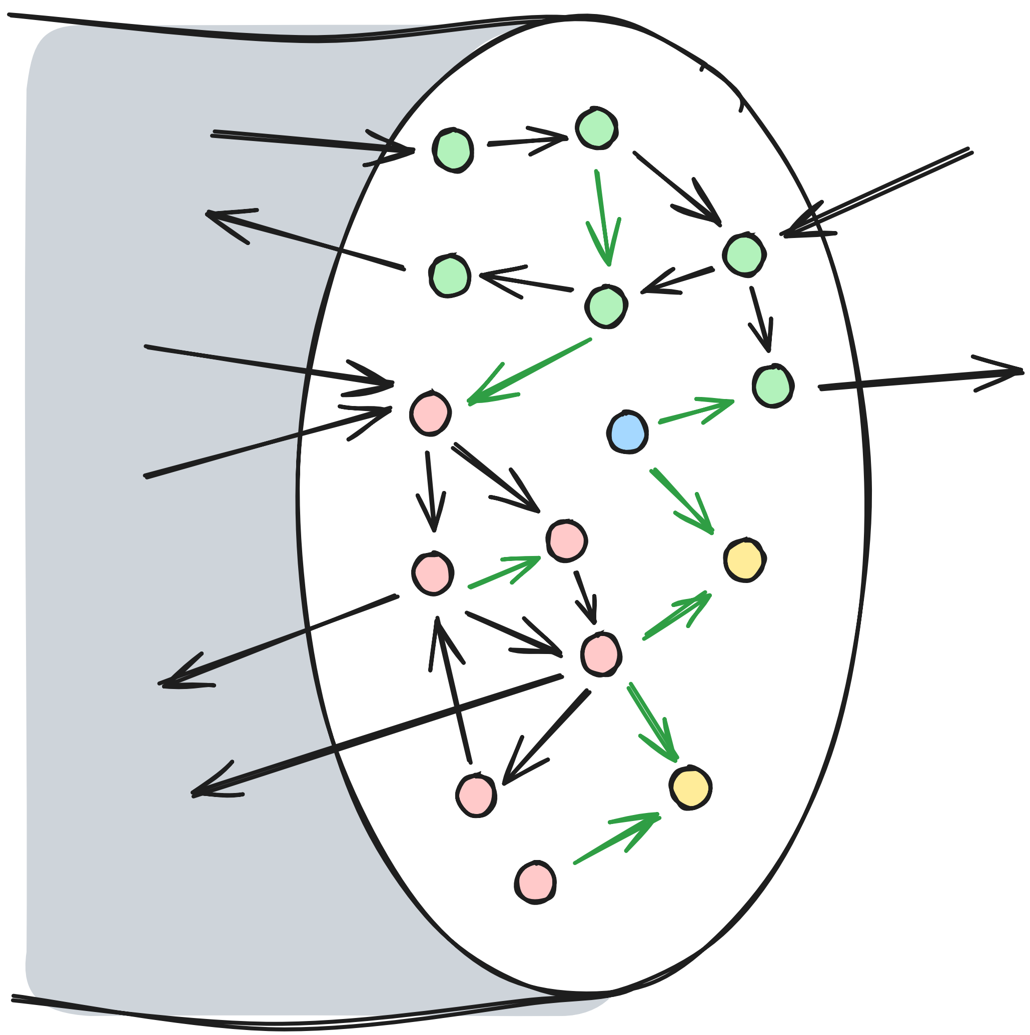
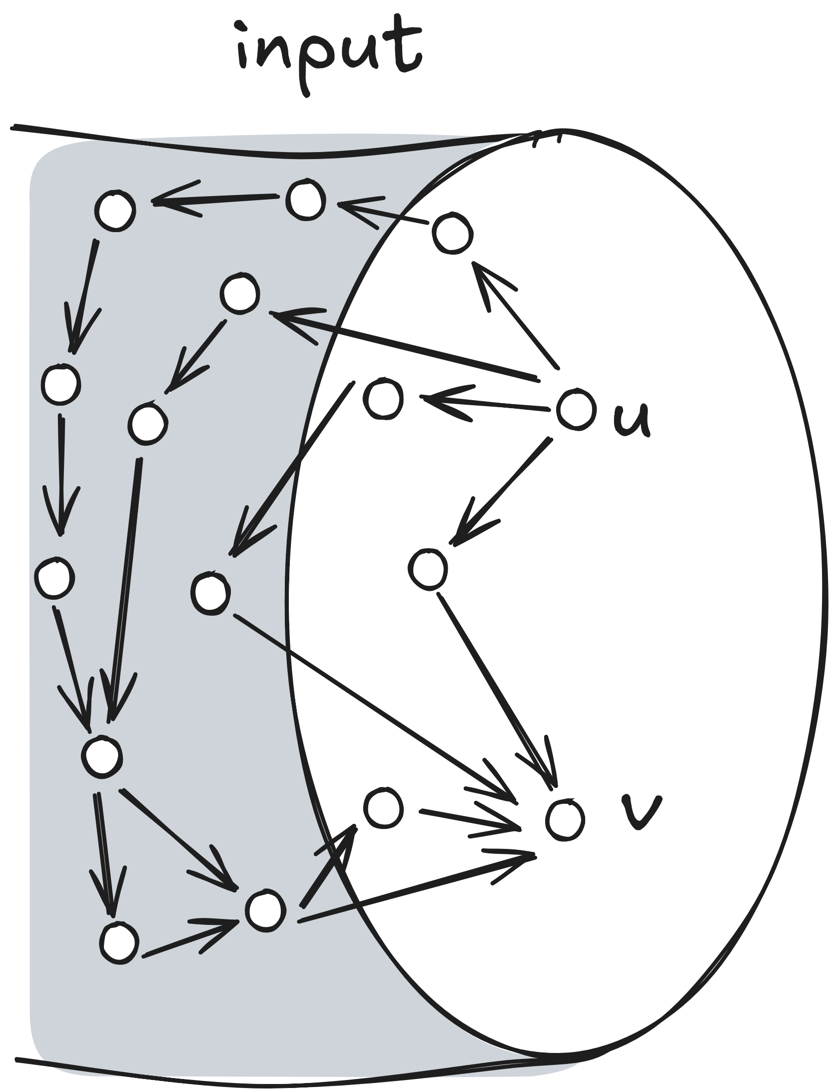
XP TREEWIDTH DYNAMIC PROGRAMMING ALGORITHM

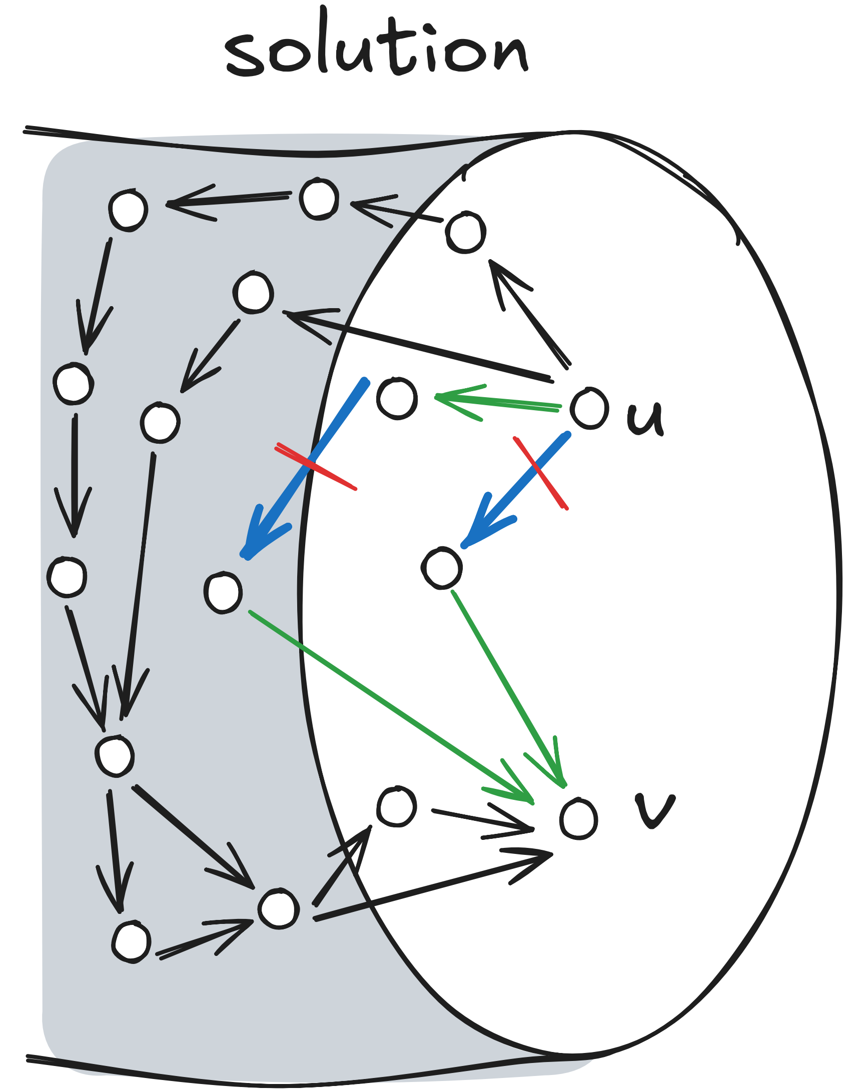


- state contains:
- reachability among vertices
    - does not exist
    - was realized
    - is realized
    - will be realized
    - complexity 4^{k^2}
- vertex balance
    - its in-degree within SCC
    - its out-degree in SCC
    - complexity n^k
- balance $\leq$ max degree: Δ^k

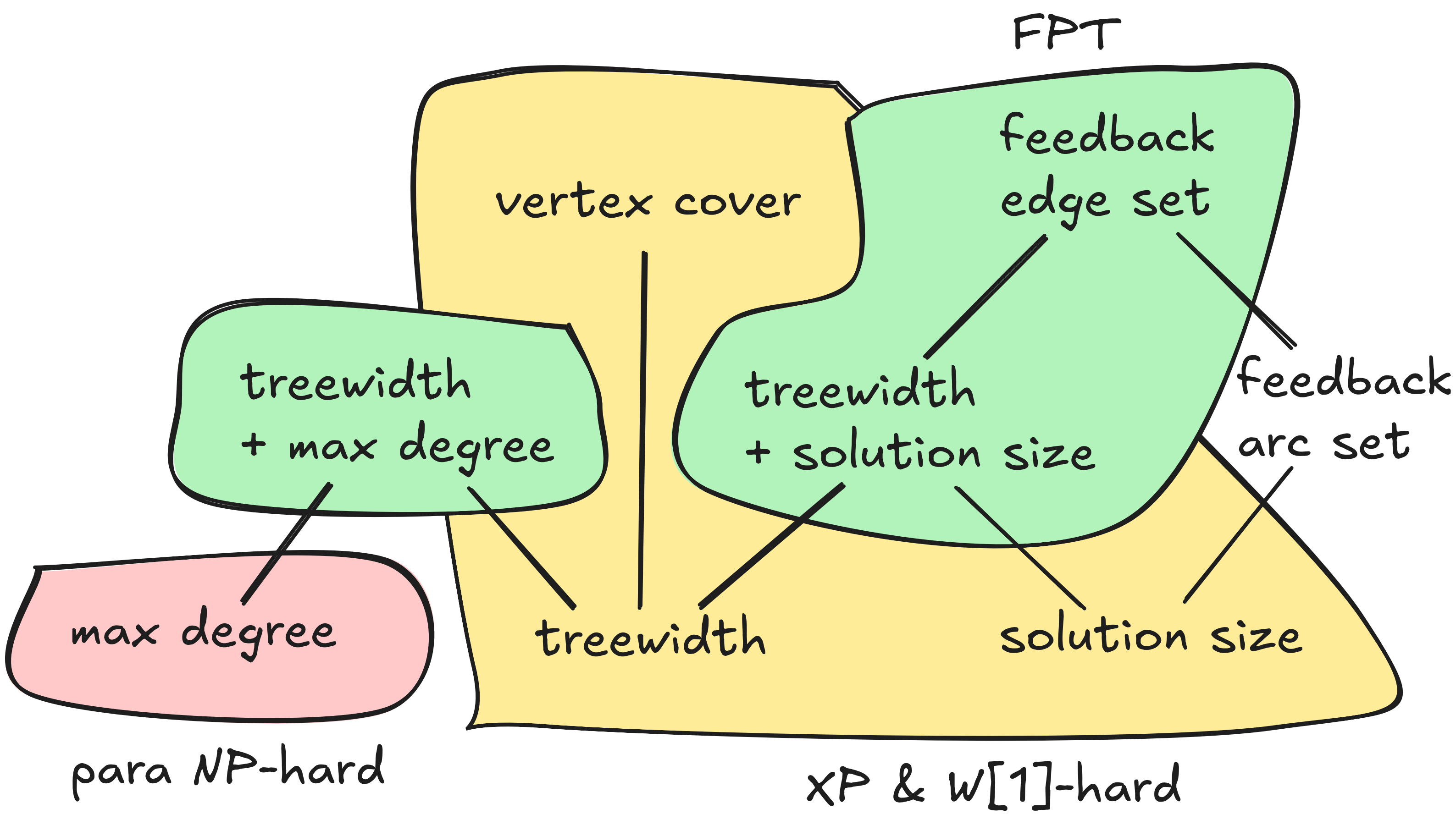
FPT BY TREEWIDTH + SOLUTION SIZE



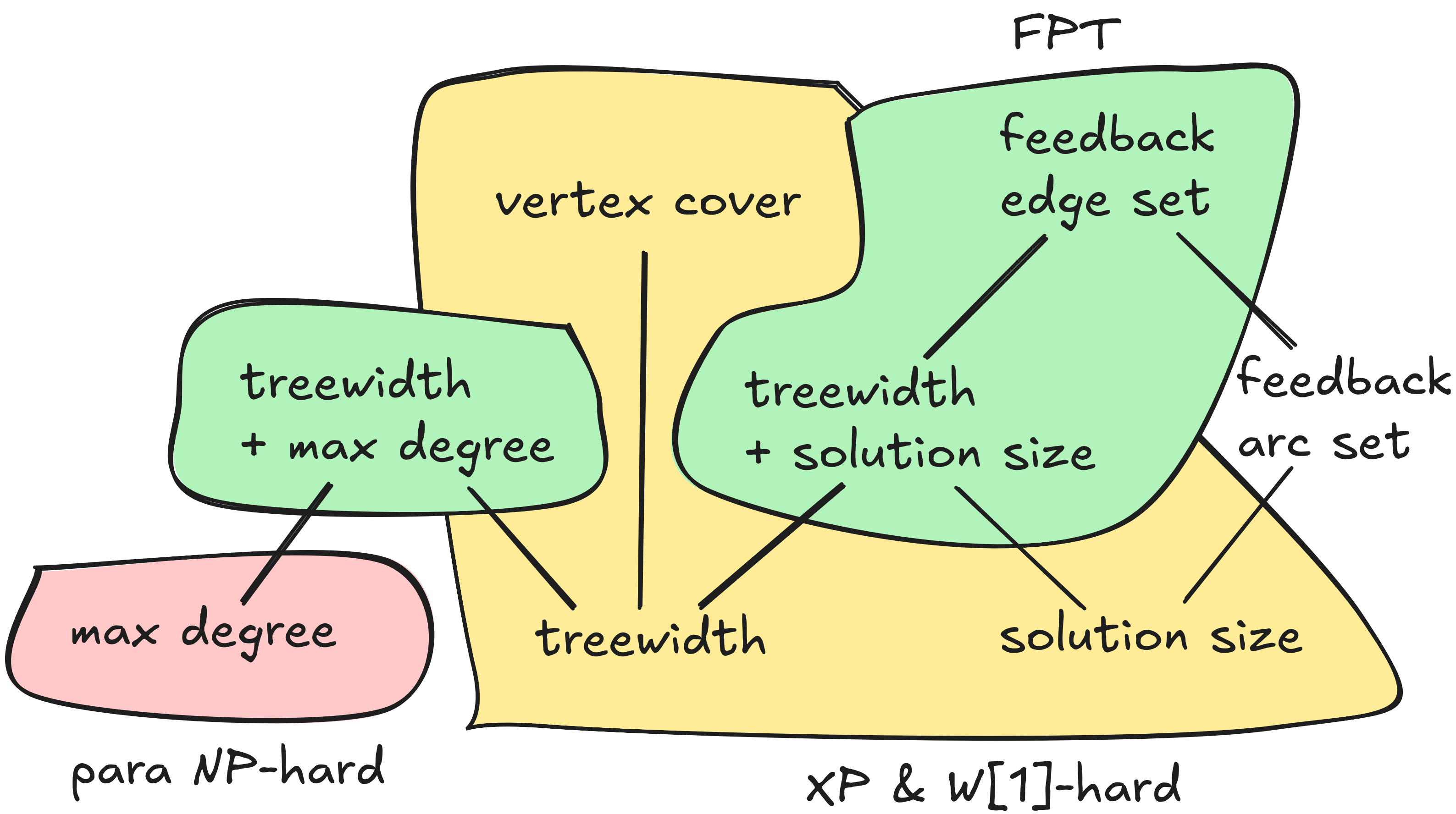
max uv flow at most 2+k



max uv flow is 2



- Design (FPT) approximation algorithms for ESCAD?
- Is ESCAD FPT with respect to Feedback Arc Set?



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Thank you!